

Wahrscheinlichkeitsrechnung (I)

Lösungen

1. (a) $\frac{1}{2}$ (b) $\frac{11}{30}$ (c) $\frac{1}{2}$ (d) $\frac{1}{6}$ (e) $\frac{7}{10}$ (f) $\frac{2}{3}$ (g) $\frac{1}{3}$

2. (a) $\frac{1}{7 \cdot 6} = \frac{1}{42}$ (b) $\frac{1}{\binom{7}{2}} = \frac{1}{21}$

3. $\frac{2 \cdot 2}{8 \cdot 7 \cdot 6 \cdot 5} = \frac{1}{420}$

4. $\frac{18}{36} = \frac{1}{2}$

5. $\frac{15}{36} = \frac{5}{12}$

6. $\frac{1}{2}$

(Wenn für die gewürfelten Zahlen $a + b + c > 10$ gilt, dann gilt für die verdeckten Augenzahlen $(7 - a) + (7 - b) + (7 - c) = 21 - (a + b + c) \leq 10$.)

7. (a) $\frac{1}{6^3} = \frac{1}{216}$ (b) $\frac{3 \cdot 5^2}{6^3} = \frac{25}{72}$ (c) $\frac{3 \cdot 5}{6^3} = \frac{5}{72}$

(d) $\frac{25}{72} + \frac{5}{72} + \frac{1}{216} = \frac{91}{216}$

8. (a) i. $1 - \frac{2^3}{3^3} = \frac{19}{27}$ ii. $\frac{3 \cdot 2^2}{3^3} = \frac{4}{9}$ iii. $\frac{2^3}{3^3} + \frac{4}{9} = \frac{20}{27}$

(b) $\frac{99^n}{100^n} = \left(\frac{99}{100}\right)^n < 0.25$, also $n \geq 138$

9. $\frac{\binom{16}{8} \cdot \binom{16}{8}}{\binom{32}{16}} \approx 27.6\%$

10. (a) $\frac{1}{9} + \frac{1}{9} = \frac{2}{9}$ (b) $\frac{1}{2} + \frac{1}{4} - \frac{1}{4} = \frac{1}{2}$ (c) $\frac{1}{2} + \frac{20}{36} - \frac{10}{36} = \frac{7}{9}$

(d) $\frac{12}{36} + \frac{1}{4} - \frac{3}{36} = \frac{1}{2}$

11. $1 - \frac{36 \cdot 32 \cdot 28}{36 \cdot 35 \cdot 34} \approx 24.7\%$

12. (a) $\frac{4 \cdot \left[2 \cdot \binom{30}{4} + 3 \cdot \binom{29}{4}\right]}{\binom{36}{9}} \approx 0.536\%$ (b) $\frac{13 \cdot \binom{4}{3} \cdot 12 \cdot \binom{4}{2}}{\binom{52}{5}} \approx 0.144\%$

13. (a) $\frac{2 \cdot (N!)^2}{(2N)!} \approx \frac{2\sqrt{\pi N}}{4^N} \approx 10^{-600}$

14. $P(k) = \frac{\binom{4}{k} \cdot \binom{8}{3-k}}{\binom{12}{3}}$; $P(0) = \frac{14}{55}$, $P(1) = \frac{28}{55}$, $P(2) = \frac{12}{55}$, $P(3) = \frac{1}{55}$

15. (a) $1 - \frac{\binom{95}{5}}{\binom{100}{5}} \approx 23.0\%$ (b) $1 - \left[\frac{\binom{95}{5}}{\binom{100}{5}} + \frac{\binom{5}{1} \cdot \binom{95}{4}}{\binom{100}{5}}\right] \approx 1.90\%$