1. a) Symmetrien: keine

hebbare Singularitäten: keine

Nullstellen: $2x^3-6x^2+6x-2 = 2(x^3-3x^2+3x-1) = 2(x-1)^3$,
also $x_1 = 1$, dreifach

Polstellen: $x_2 = 3$, zweiter Ordnung

Asymptote: $(2x^3-6x^2+6x-2):(9x^2-54x+81) = \frac{2}{9}x+\frac{2}{3}+\frac{8(3x-7)}{9(x-3)^2}$

$$\begin{array}{l} \underline{2x^3-12x^2+18x} \\ \quad 6x^2-12x \\ \quad \underline{6x^2-36x+54} \\ \qquad 24x-56 \end{array}$$

Ableitungen: $f(x) = \frac{2(x-1)^3}{9(x-3)^2}$

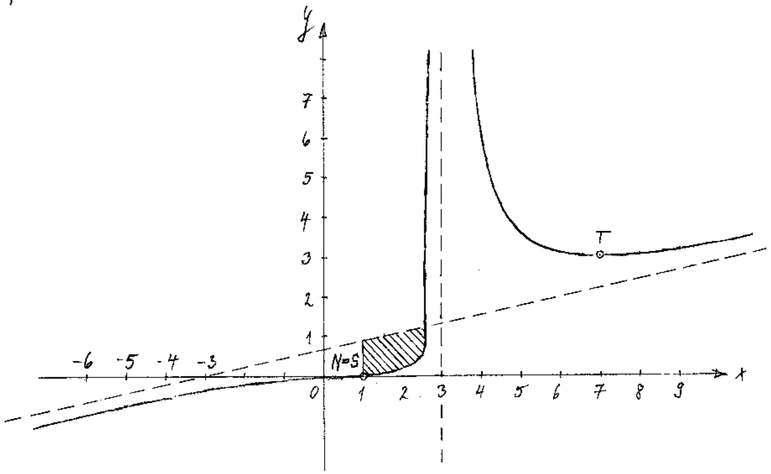
$$f'(x) = \frac{6(x-1)^2\cdot 9(x-3)^2 - 2(x-1)^3\cdot 18(x-3)}{81(x-3)^4} = \frac{2(x-1)^2(x-7)}{9(x-3)^3}$$

$$f''(x) = \frac{2[2(x-1)(x-7)+(x-1)^2]\cdot 9(x-3)^3 - 2(x-1)^2(x-7)\cdot 27(x-3)^2}{81(x-3)^6} = \frac{16(x-1)}{3(x-3)^4}$$

Extrema: $x_3 = 7$, $y_3 = 3$, $f''(x_3) > 0$, also Tiefpunkt $T(7|3)$

Wendepunkte: $x_4 = 1$, $y_4 = 0$ ist – als dreifache NS – ein Sattelpunkt $S(1|0)$, $m_t = 0$

Graph:

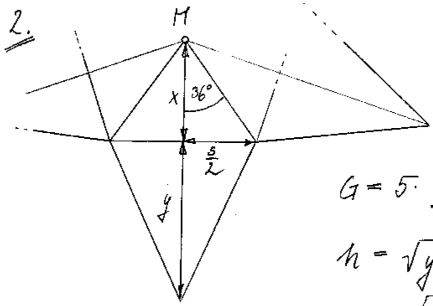


b) mit der Substitution $u = x-3$:

$$A = -\frac{8}{9}\int_{-2}^{-\frac{2}{3}} \frac{3u+2}{u^2}\cdot du = -\frac{8}{9}\left(\int_{-2}^{-\frac{2}{3}} \frac{3}{u}\,du + \int_{-2}^{-\frac{2}{3}} \frac{2}{u^2}\,du\right)$$

$$= -\frac{8}{9}\left[3\ln|u| - 2\cdot\frac{1}{u}\right]_{-2}^{-\frac{2}{3}} = \frac{8}{9}(3\ln 3 - 2) \approx 1.15$$

2.



$$x = \frac{s}{2\tan\frac{\pi}{5}}$$

$$y = 1 - \frac{s}{2\tan\frac{\pi}{5}}$$

$$G = 5\cdot\frac{s}{2}\cdot\frac{s}{2\tan\frac{\pi}{5}} = \frac{5s^2}{4\tan\frac{\pi}{5}}$$

$$h = \sqrt{y^2-x^2} = \sqrt{1-\frac{s}{\tan\frac{\pi}{5}}+\frac{s^2}{4\tan^2\frac{\pi}{5}}-\frac{s^2}{4\tan^2\frac{\pi}{5}}} = \sqrt{1-\frac{s}{\tan\frac{\pi}{5}}}$$

$$V = \frac{1}{3}Gh = \frac{5s^2}{12\tan\frac{\pi}{5}}\sqrt{1-\frac{s}{\tan\frac{\pi}{5}}}$$

$$V^2 = \frac{25s^4}{144\tan^2\frac{\pi}{5}}\left(1-\frac{s}{\tan\frac{\pi}{5}}\right) = c_1 s^4\left(1-\frac{s}{c_2}\right), \text{ wobei } c_2 = \tan\frac{\pi}{5}$$

$$= c_1 s^4 - \frac{c_1}{c_2}s^5$$

$$(V^2)' = 4c_1 s^3 - \frac{5c_1}{c_2}s^4 = s^3\left(4c_1 - \frac{5c_1}{c_2}\cdot s\right) = 0$$

$$4c_1 - \frac{5c_1}{c_2}\cdot s = 0$$

$$s = \frac{4}{5}\cdot c_2 = \frac{4}{5}\tan\frac{\pi}{5} \approx 0{,}581$$

3. a) $f'(x) = \frac{1}{2\sqrt{x}}(e^x-2)+\sqrt{x}\,e^x \overset{!}{=} 0$

$$e^x - 2 + 2x e^x = 0$$

$$g(x) := e^x(2x+1) - 2 = 0$$

$g'(x) = e^x(2x+1)+e^x\cdot 2 = e^x(2x+3)$, also $x_{n+1} = x_n - \frac{e^{x_n}(2x_n+1)-2}{e^{x_n}(2x_n+3)}$

$x_0 \approx \frac{0+\ln 2}{2} \approx 0.3$

$x_1 = x_0 - \frac{e^{x_0}(2x_0+1)-2}{e^{x_0}(2x_0+3)} \approx 0.267$

$x_2 \approx 0.2662492$

$x_3 \approx 0.2662486$, also $\underline{\underline{T(0.266/-0.359)}}$

b) $V = \pi\int_0^{\ln 2} x(e^x-2)^2\,dx = \pi\int_0^{\ln 2}(xe^{2x}-4xe^x+4x)\,dx$

- $\int_0^{\ln 2} x\cdot e^{2x}\,dx = \frac{x}{2}e^{2x}\Big|_0^{\ln 2} - \frac{1}{2}\int_0^{\ln 2} e^{2x}\,dx = 2\ln 2 - \frac{1}{4}\left[e^{2x}\right]_0^{\ln 2}$
  $= 2\ln 2 - \frac{1}{4}(4-1) = 2\ln 2 - \frac{3}{4}$
- $-4\int_0^{\ln 2} xe^x\,dx = -4\left(xe^x\Big|_0^{\ln 2} - \int_0^{\ln 2} e^x\,dx\right) = -4(2\ln 2-(2-1))$
  $= -8\ln 2 + 4$
- $\int_0^{\ln 2} 4x\,dx = \left[2x^2\right]_0^{\ln 2} = 2(\ln 2)^2$

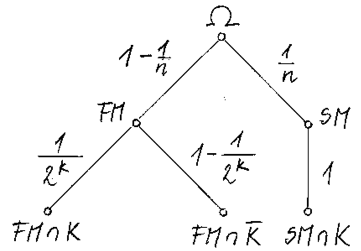
$V = \pi\left(2\ln 2 - \frac{3}{4} - 8\ln 2 + 4 + 2(\ln 2)^2\right)$
$= \pi\left(2(\ln 2)^2 - 6\ln 2 + \frac{13}{4}\right)$
$\approx \underline{\underline{0.163}}$

4. a) $\frac{10!}{3!\,2!} = \underline{\underline{302'400 \text{ Anagramme}}}$

b)

Wortanfang	Anzahl Wörter	Anfang	Anzahl
B...	$9!/(3!2!) = 30'240$	LIEBEE...	$4!/2! = 12$
E...	$9!/(2!2!) = 90'720$	LIEBEN...	$4!/2! = 12$
I...	$9!/(3!2!) = 30'240$	LIEBESE...	$3! = 6$
LB...	$8!/(3!2!) = 3360$	LIEBESNEST	1
LE...	$8!/(2!2!) = 10'080$		
LIB...	$7!/(3!2!) = 420$	Positionsnr.:	$\underline{\underline{165'091}}$

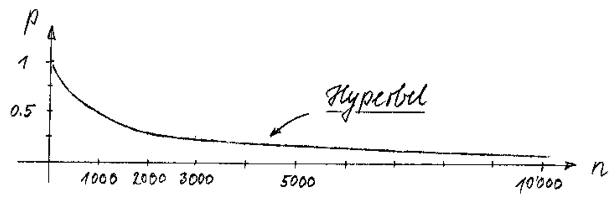
5. FM = „Die faire Münze wird ausgewählt."
SM = „Die Spezialmünze wird ausgewählt."
K = KKK...K = „Bei allen k Würfen liegt Kopf oben."



$P[SM|K] = \frac{P[SM\cap K]}{P[K]} = \frac{P[SM]}{P[K]}$
$= \frac{\frac{1}{n}}{\frac{1}{n}+\frac{n-1}{n}\cdot\frac{1}{2^k}}$
$= \frac{2^k}{2^k+n-1}$

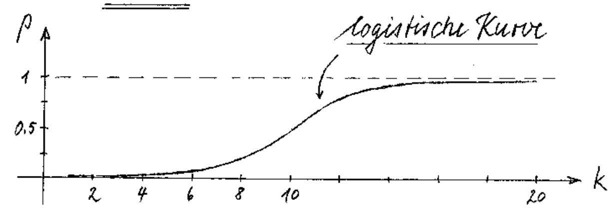
a) $P[SM|K] = \frac{1024}{1024+999} = \frac{1024}{2023} \approx \underline{\underline{50.6\%}}$

b) $P[SM|K] = \frac{1024}{1023+n} \quad (n \geq 1)$



$\frac{1024}{1023+n} \leq \frac{1}{10}$ falls $10'240 \leq 1023+n$ bzw. für $\underline{\underline{n \geq 9217}}$

c) $P[SM|K] = \frac{2^k}{2^k+999} \quad (k \geq 1)$



$\frac{2^k}{2^k+999} \geq \frac{9}{10}$, sobald $10\cdot 2^k \geq 9\cdot 2^k + 8991$
$2^k \geq 8991$
$k \geq \frac{\ln 8991}{\ln 2} \approx 13.1$, also für $\underline{\underline{k \geq 14}}$

6. a) <u>Ansatz</u>: $f(x) = ax^3 + bx^2$ (und $f'(x) = 3ax^2 + 2bx$)

$$\left.\begin{aligned} f(2) = 0: &\quad 8a + 4b = 0 \\ f'(2) = \sqrt{3}: &\quad 12a + 4b = \sqrt{3} \end{aligned}\right\} \quad 4a = \sqrt{3}, \quad a = \frac{\sqrt{3}}{4}, \quad b = -2a = -\frac{\sqrt{3}}{2}$$

<u>Lösung</u>: $\underline{\underline{f(x) = \frac{\sqrt{3}}{4}x^3 - \frac{\sqrt{3}}{2}x^2}}$

b)
$$\begin{aligned}
p_0 &= P[X^2 = 0] = P[X = 0] = \binom{4}{0}\cdot\left(\tfrac{1}{5}\right)^0\left(\tfrac{4}{5}\right)^4 = \tfrac{256}{625} = 0.4096 \\
p_1 &= P[X^2 = 1] = P[X = 1] = \binom{4}{1}\cdot\left(\tfrac{1}{5}\right)^1\left(\tfrac{4}{5}\right)^3 = \tfrac{256}{625} = 0.4096 \\
p_4 &= P[X^2 = 4] = P[X = 2] = \binom{4}{2}\cdot\left(\tfrac{1}{5}\right)^2\left(\tfrac{4}{5}\right)^2 = \tfrac{96}{625} = 0.1536 \\
p_9 &= P[X^2 = 9] = P[X = 3] = \binom{4}{3}\cdot\left(\tfrac{1}{5}\right)^3\left(\tfrac{4}{5}\right)^1 = \tfrac{16}{625} = 0.0256 \\
p_{16} &= P[X^2 = 16] = P[X = 4] = \binom{4}{4}\cdot\left(\tfrac{1}{5}\right)^4\left(\tfrac{4}{5}\right)^0 = \tfrac{1}{625} = 0.0016
\end{aligned}$$

$$E[X^2] = 0\cdot\frac{256}{625} + 1\cdot\frac{256}{625} + 4\cdot\frac{96}{625} + 9\cdot\frac{16}{625} + 16\cdot\frac{1}{625} = \underline{\frac{32}{25}} = \underline{\underline{1.28}}$$

$$\begin{aligned}
Var(X^2) &= \left(0 - \frac{32}{25}\right)^2\cdot\frac{256}{625} + \left(1 - \frac{32}{25}\right)^2\cdot\frac{256}{625} + \left(4 - \frac{32}{25}\right)^2\cdot\frac{96}{625} \\
&\quad + \left(9 - \frac{32}{25}\right)^2\cdot\frac{16}{625} + \left(16 - \frac{32}{25}\right)^2\cdot\frac{1}{625} = \underline{\frac{464}{125}} = \underline{\underline{3.712}}
\end{aligned}$$

c) <u>Teststatistik</u>: T = Anzahl der von Andrea gewonnenen Runden $\sim Bin(20, p)$

<u>Nullhypothese H_0</u>: $p = p_0 = \frac{3}{4}$

<u>Alternativen H_A</u>: $p < p_0$ (einseitig!)

<u>Verwerfungsbereich</u>: aus Tabellen:

$P_{H_0}[T \in \{0, \ldots, 11\}] = 0.0409$

$P_{H_0}[T \in \{0, \ldots, 12\}] = 0.102$, also

$K = \{0, 1, 2, \ldots, 11\}$

<u>Entscheidung</u>: $13 \notin K$, also muss H_0 <u>beibehalten</u> werden.

Andreas Behauptung könnte also stimmen.