1. $f'(x) = \frac{(3x^2-2x)(x^2-2) - (x^3-x^2+2)\cdot 2x}{(x^2-2)^2} = \frac{x^4-6x^2}{(x^2-2)^2} = \frac{x^2(x^2-6)}{(x^2-2)^2}$

$f''(x) = \frac{(4x^3-12x)(x^2-2)^{\cancel{2}} - (x^4-6x^2)\cdot 2\cancel{(x^2-2)}\cdot 2x}{(x^2-2)^{\cancel{4}\,3}} = \frac{4x^3+24x}{(x^2-2)^3} = \frac{4x(x^2+6)}{(x^2-2)^3}$

Symmetrien: keine offensichtlichen

Singularitäten / Null- und Polstellen:

Nullstelle $x_1 = -1$ durch Probieren

$(x^3-x^2+2):(x+1) = x^2-2x+2$

x^3+x^2

$-2x^2$

$-2x^2-2x$

$2x$

$2x+2$

0

$x_{2,3} = \frac{2 \pm \sqrt{4-4\cdot 1\cdot 2}}{2}$ „geht nicht"

keine Singularität

Nullstelle $x_1 = -1$, einfach

Polstelle $x_{2,3} = \pm\sqrt{2}$, je erste Ordnung

Asymptoten:

$(x^3-x^2+2):(x^2-2) = x-1+\frac{2x}{x^2-2}$

x^3-2x

$-x^2+2x$

$-x^2+2$

$2x$

schiefe Asymptote $y = x-1$

Extrema:

$x_{4,5} = \pm\sqrt{6}$, $y_{4,5} = \frac{\pm 6\sqrt{6}-4}{4} = \frac{\pm 3\sqrt{6}-2}{2}$

$f''(x_{4,5}) = \pm\frac{4\cdot\sqrt{6}\cdot 12}{4^3} = \pm\frac{3}{4}\sqrt{6} \gtrless 0$

Tiefpunkt $T(\sqrt{6} / \frac{3}{2}\sqrt{6}-1) \approx T(2{,}45/2{,}67)$

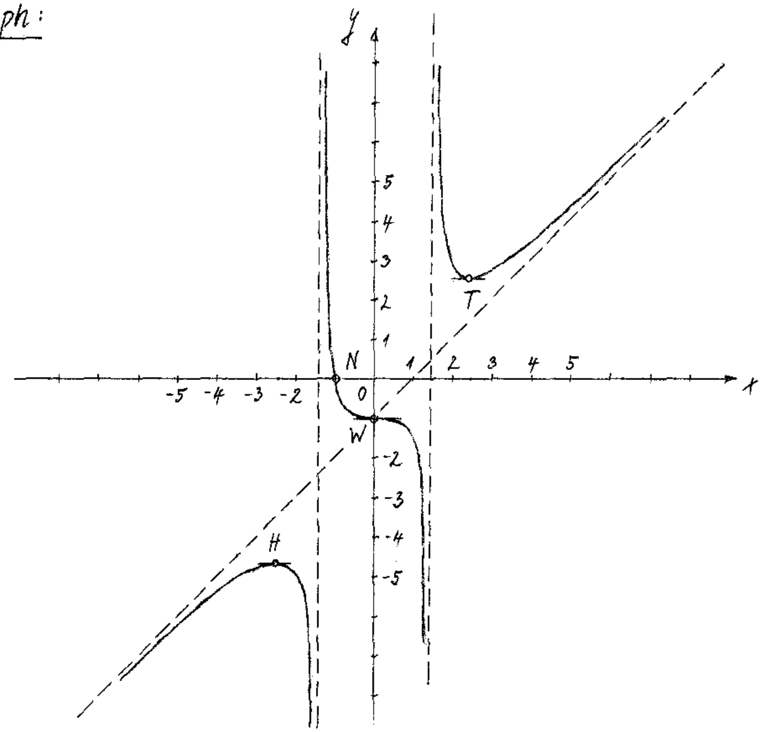
Hochpunkt $H(-\sqrt{6} / -\frac{3}{2}\sqrt{6}-1) \approx H(-2{,}45/-4{,}67)$

Wendepunkt:

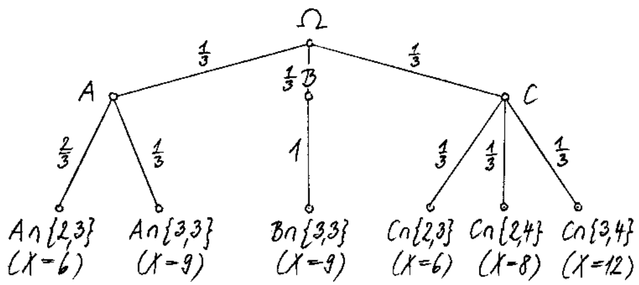
$x_6 = 0$, $y_6 = -1$, $f'(x_6) = m_t = 0$

Wendepunkt $W(0/-1)$ mit horizontaler Wendetangente (Terrassenpkt.)

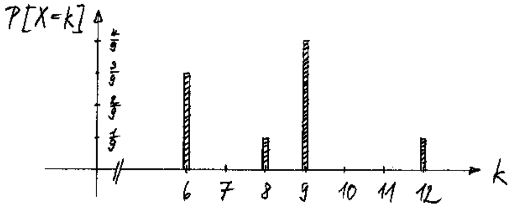
Graph:



2. a)



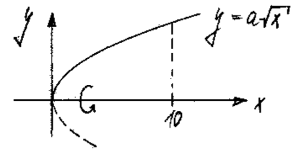
b)
$$P[X=6] = \tfrac{1}{3}\cdot\tfrac{2}{3} + \tfrac{1}{3}\cdot\tfrac{1}{3} = \tfrac{1}{3}$$
$$P[X=8] = \tfrac{1}{3}\cdot\tfrac{1}{3} = \tfrac{1}{9}$$
$$P[X=9] = \tfrac{1}{3}\cdot\tfrac{1}{3} + \tfrac{1}{3}\cdot 1 = \tfrac{4}{9}$$
$$P[X=12] = \tfrac{1}{3}\cdot\tfrac{1}{3} = \tfrac{1}{9}$$



c)
$$E[X] = \tfrac{1}{3}\cdot 6 + \tfrac{1}{9}\cdot 8 + \tfrac{4}{9}\cdot 9 + \tfrac{1}{9}\cdot 12 = \underline{\tfrac{74}{9}} \ (\approx 8.22)$$
$$Var(X) = \tfrac{1}{3}\left(6-\tfrac{74}{9}\right)^2 + \tfrac{1}{9}\left(8-\tfrac{74}{9}\right)^2 + \tfrac{4}{9}\left(9-\tfrac{74}{9}\right)^2 + \tfrac{1}{9}\left(12-\tfrac{74}{9}\right)^2$$
$$= \underline{\tfrac{284}{81}} \ (\approx 3{,}51)$$
$$\sigma(X) = \underline{\underline{\tfrac{2}{9}\cdot\sqrt{71}}} \ (\approx 1.87)$$

d)
$$P[A \mid X=6] = \frac{P[A\cap\{X=6\}]}{P[X=6]} = \frac{\frac{1}{3}\cdot\frac{2}{3}}{\frac{1}{3}\cdot\frac{2}{3}+\frac{1}{3}\cdot\frac{1}{3}} = \underline{\underline{\tfrac{2}{3}}}$$

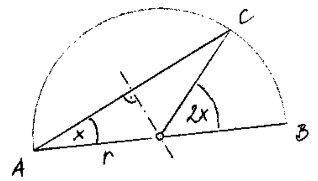
3.



a)
$$V_{rot} = \pi\cdot\int_0^{10} a^2\cdot x\,dx = a^2\pi\cdot\left[\tfrac{1}{2}x^2\right]_0^{10} = 50a^2\pi \stackrel{!}{=} 300$$
$$a^2\pi = 6$$
$$(1.38 \approx)\ \ \underline{\underline{a = \sqrt{\tfrac{6}{\pi}}}}$$

b)
$$f'(x) = \frac{a}{2\sqrt{x}}$$
$$1+f'(x)^2 = 1+\frac{a^2}{4x} = \frac{4x+a^2}{4x}$$
$$f(x)\cdot\sqrt{1+f'(x)^2} = a\sqrt{x}\cdot\sqrt{\frac{4x+a^2}{4x}} = \frac{a}{2}\sqrt{4x+a^2}$$
$$S = 2\pi\int_0^{10}\frac{a}{2}\sqrt{4x+a^2}\,dx = a\pi\cdot\int_0^{10}\sqrt{4x+a^2}\,dx$$
$$= a\pi\cdot\int_{a^2}^{40+a^2}\sqrt{u}\cdot\frac{1}{4}\,du = \frac{a\pi}{4}\cdot\left[\tfrac{2}{3}u^{\frac{3}{2}}\right]_{a^2}^{40+a^2}$$
$$= \frac{2a\pi}{12}\left[(a^2+40)^{\frac{3}{2}} - a^3\right] = \frac{\pi}{6}\cdot\sqrt{\frac{6}{\pi}}\left[\left(\frac{6}{\pi}+40\right)^{\frac{3}{2}} - \left(\frac{6}{\pi}\right)^{\frac{3}{2}}\right]$$
$$= \sqrt{\frac{\pi}{6}}\cdot\left(\frac{2}{\pi}\right)^{\frac{3}{2}}\cdot\left[(3+20\pi)^{\frac{3}{2}} - 3^{\frac{3}{2}}\right]$$
$$= \frac{2}{\sqrt{3}\pi}\cdot\left[(3+20\pi)^{\frac{3}{2}} - 3^{\frac{3}{2}}\right] = \underline{\underline{\frac{2}{3\pi}\left[\sqrt{3}(3+20\pi)^{\frac{3}{2}} - 9\right]}}$$
$$(\approx 194\ cm^2)$$

4.



$$\overline{AC} = 2\cdot r\cos x$$
$$\overset{\frown}{ABC} = 2r + r\cdot 2x$$

$$2r + 2rx = 3\cdot 2r\cos x$$
$$1 + x = 3\cos x$$
$$3\cos x - x - 1 = 0$$

$$x_0 = \frac{\pi}{6} \;(\hat{=}\, 30^\circ)$$

$$x_{n+1} = x_n - \frac{3\cos x_n - x_n - 1}{-3\sin x_n - 1}$$

$$x_1 \approx 0.953$$
$$x_2 \approx 0.891$$
$$x_3 \approx 0.889(47) \,\hat{=}\, 50.96^\circ \approx \underline{\underline{51^\circ}}$$

5. a) X = Anzahl nicht abgeholter Karten $\sim Bin(25, \frac{1}{10})$

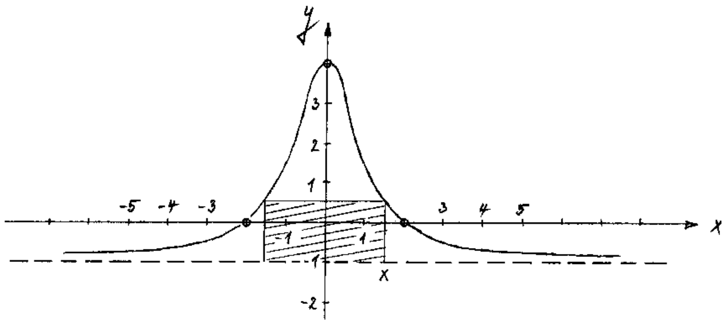
$$P[X=0] = \binom{25}{0}\cdot\left(\frac{1}{10}\right)^0\cdot\left(1-\frac{1}{10}\right)^{25-0}$$
$$= \left(\frac{9}{10}\right)^{25}$$
$$\approx \underline{\underline{7.18\%}}$$

b) $$P[X \geq 3] = 1 - P[X \leq 2]$$
$$= 1 - \binom{25}{0}\left(\frac{1}{10}\right)^0\left(\frac{9}{10}\right)^{25} - \binom{25}{1}\left(\frac{1}{10}\right)^1\left(\frac{9}{10}\right)^{24} - \binom{25}{2}\left(\frac{1}{10}\right)^2\left(\frac{9}{10}\right)^{23}$$
$$\approx 1 - 0.537$$
$$\approx \underline{\underline{46.3\%}}$$

c) $X \sim Bin(27, \frac{1}{10})$

$$P[X \leq 1] = \binom{27}{0}\left(\frac{1}{10}\right)^0\left(\frac{9}{10}\right)^{27} + \binom{27}{1}\left(\frac{1}{10}\right)^1\left(\frac{9}{10}\right)^{26}$$
$$\approx \underline{\underline{23.3\%}}$$

6. a) $f(x) = \frac{-(x-2)(x+2)}{x^2+1} = \underline{\underline{\frac{-x^2+4}{x^2+1}}} \quad (a=-1,\ b=0,\ c=4)$



b) $(-x^2+4):(x^2+1) = -1 + \frac{5}{x^2+1}$

$\underline{-x^2-1}$

5

$$A = 2\cdot\int_0^2 f(x)\,dx = 2\cdot\int_0^2\left(-1+5\cdot\frac{1}{x^2+1}\right)dx$$

$$= 2\cdot\Big[-x+5\cdot\arctan x\Big]_0^2 = \underline{\underline{10\arctan 2 - 4}}$$

(≈ 7.07)

c) $A = 2x\cdot(f(x)+1)$

$= \frac{10x}{x^2+1}$

$$A' = \frac{10(x^2+1) - 10x\cdot 2x}{(x^2+1)^2} = \frac{-10x^2+10}{(x^2+1)^2} \overset{!}{=} 0$$

$x^2 = 1$

$x = \cancel{\pm} 1$

$$A_{max} = \frac{10\cdot 1}{1^2+1} = \underline{\underline{5}}$$