1. Kurvendiskussion

a) Symmetrie: gerade Fkt. (achsensymm. bezgl. y-Achse)

hebbare Singularitäten: keine

Nullstellen: $f(x) = \frac{(x^2+3)(x^2-1)}{4x^2}$, also $x_{1,2} = \pm 1$ (je einfach)

Polstellen: $x_3 = 0$ (zweiter Ordng.)

Asymptoten: senkrechte Asymptote bei $x=0$

$f(x) = \frac{1}{4}x^2 + \frac{1}{2} - \frac{3}{4x^2}$, also krummlinige Asymptote $y = \frac{1}{4}x^2 + \frac{1}{2}$

Ableitungen: $f'(x) = \frac{1}{2}x + \frac{3}{2x^3} = \frac{x^4+3}{2x^3}$

$f''(x) = \frac{1}{2} - \frac{9}{2x^4} = \frac{x^4-9}{2x^4}$

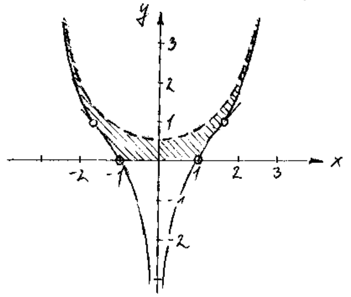
$f'''(x) = \frac{18}{x^5}$

Extrema: keine

Wendepunkte: $x^4 - 9 = 0$, $x_{4,5} = \pm\sqrt{3}$, $y_{4,5} = \frac{3}{4} + \frac{1}{2} - \frac{1}{4} = 1$, $f'(x_{3,4}) = \frac{12}{\pm 2\cdot 3\sqrt{3}} = \pm\frac{2}{3}\sqrt{3} \approx \pm 1.15$

$f''(x_{3,4}) \neq 0$

Graph:



b) $A = 2\cdot\int_0^1 g(x)\,dx + 2\int_1^\infty (g(x)-f(x))\,dx$

$= 2\int_0^1 \left(\frac{1}{4}x^2+\frac{1}{2}\right)dx + 2\int_1^\infty \frac{3}{4}x^{-2}\,dx$

$= 2\left(\frac{1}{4}\cdot\frac{1}{3}\cdot 1^3 + \frac{1}{2}\cdot 1\right) + 2\cdot\left(0 - \frac{3}{4}\cdot(-1)\cdot\frac{1}{1}\right)$

$= \underline{\underline{\frac{8}{3}}}$

c) $\frac{x^4+3}{2x^3} = \frac{3}{2}$, $2x^4+6 = 6x^3$, $x^4 - 3x^3 + 3 = 0$

$x_0 = 2$

$x_{n+1} = x_n - \frac{x_n^4 - 3x_n^3 + 3}{4x_n^3 - 9x_n^2}$, $x_1 = \frac{3}{4}$, $x_2 \approx 1.358$, $x_3 \approx 1.189$, $x_4 \approx \underline{\underline{1.182}}$

(zweite Lösung: $\underline{\underline{2.87}}$)

2. Abstände

a) $g: \vec{r} = \begin{pmatrix}1\\9\\0\end{pmatrix} + t\begin{pmatrix}2\\-2\\-1\end{pmatrix}$

$\vec{AB}\times\vec{AC} = \begin{pmatrix}2\\-2\\-1\end{pmatrix}\times\begin{pmatrix}-2\\-10\\-2\end{pmatrix} = \begin{pmatrix}-6\\6\\-24\end{pmatrix} = (-6)\cdot\begin{pmatrix}1\\-1\\4\end{pmatrix}$

$E: x - y + 4z + d = 0$, $A\in E$: $1 - 9 + 4\cdot 0 + d = 0$

$d = 8$, $E: x - y + 4z + 8 = 0$

b) $HNF(E): \frac{x-y+4z+8}{\sqrt{18}} = 0$

$\overline{DE} = \left|\frac{7-3+24+8}{\sqrt{18}}\right| = 6\sqrt{2} \approx \underline{\underline{8.49}}$

c) $\vec{CP} = \begin{pmatrix}2+2t\\10-2t\\2-t\end{pmatrix}$, $\overline{CP} = \sqrt{9t^2-36t+108}$

$\vec{DP} = \begin{pmatrix}-6+2t\\6-2t\\-6-t\end{pmatrix}$, $\overline{DP} = \sqrt{9t^2-36t+108}$ $\Big\}$ = für alle $t\in\mathbb{R}$

d) $\vec{CP}\cdot\vec{DP} = (2+2t)(-6+2t)+(10-2t)(6-2t)+(2-t)(-6-t) = 9t^2-36t+36 = 0$

$t^2 - 4t + 4 = 0$

$(t-2)^2 = 0$

$t = 2$: $\underline{\underline{P(5/5/-2)}}$

3. Trigonometrie

a) $\overline{CT} : \sin\frac{\alpha}{2} = w_\alpha : \sin\gamma$

$\overline{TB} : \sin\frac{\alpha}{2} = w_\alpha : \sin\beta \quad \Big| :$

$\overline{CT} : \overline{TB} = \sin\beta : \sin\gamma$ ■

b) 1) $\alpha = 180° - 60° - 45° = 75°$, $\frac{\alpha}{2} = 37.5°$

2) $\overline{CT} : \sin\frac{\alpha}{2} = w_\alpha : \sin\gamma \rightsquigarrow \overline{CT} = w_\alpha\cdot\frac{\sin\frac{\alpha}{2}}{\sin\gamma} \approx 5.62$ cm

$\overline{TB} : \sin\frac{\alpha}{2} = w_\alpha : \sin\beta \rightsquigarrow \overline{TB} = w_\alpha\cdot\frac{\sin\frac{\alpha}{2}}{\sin\beta} \approx 6.89$ cm

3) $\overline{BC} = \overline{CT} + \overline{TB} \approx 12.5$ cm

$\overline{BM} = \frac{1}{2}\overline{BC} \approx 6.26$ cm

4) $\overline{AB} : \sin\gamma = \overline{BC} : \sin\alpha \rightsquigarrow \overline{AB} = \overline{BC}\cdot\frac{\sin\gamma}{\sin\alpha} \approx 11.2$ cm

5) $s_a = \overline{AM} = \sqrt{\overline{AB}^2 + \overline{BM}^2 - 2\overline{AB}\cdot\overline{BM}\cdot\cos\beta} \approx \underline{\underline{8.11\text{ cm}}}$

4. Reflexion an der Kugel

a) $x^2 + 8x + 16 + y^2 - 6y + 9 + z^2 - 10z + 25 = 31 + 16 + 9 + 25$

$K: (x+4)^2 + (y-3)^2 + (z-5)^2 = 9^2 \rightsquigarrow M(-4/3/5)$; $r = 9$

b) $LP: \vec{r} = \begin{pmatrix}20\\24\\8\end{pmatrix} + t\begin{pmatrix}-2\\-2\\-1\end{pmatrix}$

$LP\cap K: (24-2t)^2 + (21-2t)^2 + (3-t)^2 = 81$

$9t^2 - 186t + 1026 = 81$

$3t^2 - 62t + 315 = 0$

$t_{1,2} = \frac{62\pm\sqrt{62^2 - 4\cdot 3\cdot 315}}{6} = \frac{62\pm 8}{6}$

$\underline{\underline{R(2/6/-1)}} \leftsquigarrow t_1 = 9$

$\left(t_2 = \frac{35}{3} > t_1\right)$

c) $\vec{MR} = \begin{pmatrix}6\\3\\-6\end{pmatrix} = 3\cdot\begin{pmatrix}2\\1\\-2\end{pmatrix}$

Tangentialebene E in R: $2x + y - 2z + d = 0$

$R\in E$: $2\cdot 2 + 6 - 2(-1) + d = 0$

$d = -12$

$E: 2x + y - 2z - 12 = 0$

Lot ℓ b.E durch L: $\vec{r} = \begin{pmatrix}20\\24\\8\end{pmatrix} + t\begin{pmatrix}2\\1\\-2\end{pmatrix}$

$\ell\cap E: 2(20+2t) + (24+t) - 2(8-2t) - 12 = 0$

$9t + 36 = 0$

$t = -4$

$2t = -8 \rightsquigarrow L'(4/16/24)$

$\underline{\underline{\ell' = L'R: \vec{r} = \begin{pmatrix}4\\16\\24\end{pmatrix} + t\begin{pmatrix}-2\\-10\\-25\end{pmatrix}}}$

d) $\varepsilon = \arccos\frac{\vec{L'R}\cdot\vec{RL}}{|\vec{L'R}|\cdot|\vec{RL}|} = \arccos\frac{\begin{pmatrix}-2\\-10\\-25\end{pmatrix}\cdot\begin{pmatrix}18\\18\\9\end{pmatrix}}{729} = \arccos\left(-\frac{49}{81}\right) \approx \underline{\underline{127°}}$

5. Rotationskörper

a) Nullstellen: $x_1 = 0,\ x_2 = 1$

Maximum: $\frac{1}{2\sqrt{\ldots}} \cdot \left[x(1-x)e^x\right]' = 0$

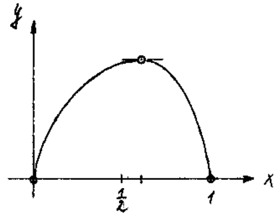
$$\left[(x-x^2)e^x\right]' = 0$$

$$(1-2x)e^x + (x-x^2)e^x = 0 \quad | : e^x$$

$$x^2 + x - 1 = 0$$

$$x = \frac{-1 \pm \sqrt{1+4}}{2} = \frac{-1+\sqrt{5}}{2} \approx 0.618,\ y \approx 0.662$$

b)



c) $V = \pi \cdot \int_0^1 x(1-x)e^x\,dx$

$$= \pi \cdot \left[\int_0^1 xe^x dx - \int_0^1 x^2 e^x dx\right]$$

$$= \pi \cdot \left[\int_0^1 xe^x dx - x^2 e^x\Big|_0^1 + 2\int_0^1 xe^x dx\right]$$

$$= \pi \cdot \left[3\int_0^1 xe^x dx - e\right]$$

$$= \pi \cdot \left[3\left(e - \int_0^1 e^x dx\right) - e\right]$$

$$= \pi\left[3(e-(e-1)) - e\right]$$

$$= \underline{\pi \cdot (3-e)} \approx 0.885$$

6. Kurzaufgaben

a) $f(x) = \underline{\frac{(x+4)(x-2)(x-4)}{4(x+2)}}$

b) $f(x) = ax^3 + bx^2 + cx + d$

I $f(0) = 0: \ d = 0$

II $f'(0) = 0: \ c = 0$

III $f(3) = 0: \ 27a + 9b = 0$

IV $f'(3) = 1: \ 27a + 6b = 1 \quad | -$

$3b = -1,\ b = -\frac{1}{3},\ a = \frac{1}{9},\ f(x) = \underline{\frac{1}{9}x^3 - \frac{1}{3}x^2}$

c) $\int_1^{\sqrt{3}} \frac{2x}{\sqrt{x^2+1}} dx = \int_2^4 \frac{1}{\sqrt{u}} du = 2\sqrt{u}\Big|_2^4 = \underline{4 - 2\sqrt{2}} \approx 1.17$

d) $y' = \frac{e^x - e^{-x}}{2}$

$(y')^2 = \frac{e^{2x} - 2 + e^{-2x}}{4}$

$1 + (y')^2 = \frac{e^{2x} + 2 + e^{-2x}}{4} = \left(\frac{e^x + e^{-x}}{2}\right)^2$

$l = \int_{-1}^{1} \sqrt{1 + (y')^2}\,dx = \frac{1}{2}\int_{-1}^{1} (e^x + e^{-x})\,dx = \frac{1}{2}\left[e^x - e^{-x}\right]_{-1}^{1} = \underline{e - \frac{1}{e}} \approx 2.35$