1. Kurvendiskussion

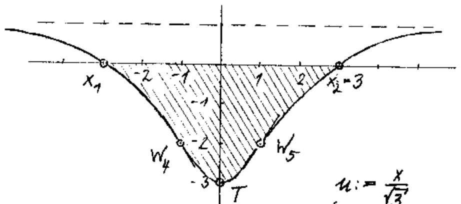
a) Symmetrie: gerade Fkt. (achsensymm. bzgl. y-Achse)
hebbare Singularitäten: keine
Nullstellen: $x_{1,2} = \pm 3$, je einfach
Polstellen: keine
Asymptoten: $f(x) = 1 - \frac{12}{x^2+3}$, also waagrechte Asymptote $y = 1$
Ableitungen: $f'(x) = \frac{24x}{(x^2+3)^2}$

$$f''(x) = -\frac{72(x^2-1)}{(x^2+3)^3}$$

$$f'''(x) = \frac{288x(x^2-3)}{(x^2+3)^4}$$

Extrema: $x_3 = 0,\ y_3 = \frac{-9}{3} = -3,\ f''(0) = \frac{8}{3} > 0$: $T(0/-3)$
Wendepkte: $x_{4,5} = \pm 1,\ y_{4,5} = \frac{-8}{4} = -2,\ m_{4,5} = \pm\frac{24}{16} = \pm\frac{3}{2}$:
$W_4(-1/-2)$ mit $m_4 = -\frac{3}{2}$
$W_5(1/-2)$ mit $m_5 = \frac{3}{2}$

Graph:



b) $$2\int_0^3 \left(1-\frac{12}{x^2+3}\right)dx = 2\int_0^3 \left(1-\frac{4}{\frac{x^2}{3}+1}\right)dx \overset{u:=\frac{x}{\sqrt{3}}}{=} 2x\Big|_0^3 - 8\int_0^{\sqrt{3}} \frac{1}{u^2+1}\sqrt{3}\,du$$

$$= 6 - 8\sqrt{3}\cdot\arctan u\Big|_0^{\sqrt{3}} = 6 - 8\sqrt{3}\cdot\frac{\pi}{3} = 6 - \frac{8\pi\sqrt{3}}{3} \approx -8.51$$

2. Exponentialfunktionen

a) $f(x) = x\cdot e^{1-x} - \frac{1}{e}$
$f'(x) = (1-x)e^{1-x}$
$x_0 = 0$, $x_{n+1} = x_n - \frac{x_n e^{1-x_n} - \frac{1}{e}}{(1-x_n)e^{1-x_n}}$
$x_1 = 0 - \frac{0-\frac{1}{e}}{1\cdot e} = \frac{1}{e^2} \approx 0.135$
$x_2 \approx 0.158$
$x_3 \approx 0.158 \approx 0.16$ (2te Lös. ist $3.146 \approx 3.15$)

b) $$\int_0^3 \underset{\downarrow}{x}\,\underset{\uparrow}{e^{1-x}}\,dx = -xe^{1-x}\Big|_0^3 + \int_0^3 e^{1-x}dx = -3\cdot\frac{1}{e^2} - e^{1-x}\Big|_0^3 = -\frac{3}{e^2} - \frac{1}{e^2} + e$$

$$= e - \frac{4}{e^2} \approx 2.18$$

c) $$\pi\int_0^\infty \underset{\downarrow}{x^2}\,\underset{\uparrow}{e^{2-2x}}\,dx = \pi x^2 e^{2-2x}\left(-\tfrac{1}{2}\right)\Big|_0^\infty - \pi\int_0^\infty 2x e^{2-2x}\left(-\tfrac{1}{2}\right)dx = 0 + \pi\int_0^\infty \underset{\downarrow}{x}\,\underset{\uparrow}{e^{2-2x}}\,dx$$

$$= \pi x e^{2-2x}\left(-\tfrac{1}{2}\right)\Big|_0^\infty - \pi\int_0^\infty e^{2-2x}\left(-\tfrac{1}{2}\right)dx = 0 + \frac{\pi}{2}\int_0^\infty e^{2-2x}dx$$

$$= \frac{\pi}{2} e^{2-2x}\cdot\left(-\tfrac{1}{2}\right)\Big|_0^\infty = \frac{\pi e^2}{4} \approx 5.80$$

d) $\int \frac{y'}{y}dx = \int e^{1-x}dx$
$\int \frac{1}{y}dy = \int e^{1-x}dx$
$\log y = -e^{1-x} + \tilde{c}$
$y = c\cdot e^{-e^{1-x}}$ $(c>0)$

$P(1/e)$: $e = c\cdot e^{-1}$
$c = e^2$

also $y = e^{2-e^{1-x}}$

3. Vektorgeometrie

a) $\vec{r}_D = \vec{r}_C - \vec{AB} = \begin{pmatrix}-3\\5\\-4\end{pmatrix} - \begin{pmatrix}-8\\-8\\4\end{pmatrix} = \begin{pmatrix}5\\13\\-8\end{pmatrix}$, $D(5/13/-8)$

b) $|\vec{AB}| = \sqrt{64+64+16} = \sqrt{144} = 12$
$h = \frac{3V}{|\vec{AB}|^2} = \frac{3\cdot 720}{144} = 15$
$M = M_{AC}(5/7/-2)$
Achsenrichtung $\vec{n} = \vec{AB}\times\vec{BC} = \begin{pmatrix}-8\\-8\\4\end{pmatrix}\times\begin{pmatrix}-8\\4\\-8\end{pmatrix} = \begin{pmatrix}48\\-96\\-96\end{pmatrix} = 48\begin{pmatrix}1\\-2\\-2\end{pmatrix}$
$\left|\begin{pmatrix}1\\-2\\-2\end{pmatrix}\right| = \sqrt{9} = 3$, also
$\vec{r}_S = \vec{r}_M \pm 5\begin{pmatrix}1\\-2\\-2\end{pmatrix}$: $S_1(10/-3/-12)$, $S_2(0/17/8)$

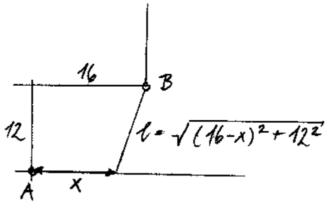
c) $\vec{AS} = \begin{pmatrix}-3\\-12\\-12\end{pmatrix} = 3\cdot\begin{pmatrix}-1\\-4\\-4\end{pmatrix}$, $\left|\begin{pmatrix}-1\\-4\\-4\end{pmatrix}\right| = \sqrt{33}$

$$\arcsin\frac{\begin{pmatrix}1\\-2\\-2\end{pmatrix}\cdot\begin{pmatrix}-1\\-4\\-4\end{pmatrix}}{3\cdot\sqrt{33}} = \arcsin\frac{15}{3\sqrt{33}} = \arcsin\frac{5}{\sqrt{33}} \approx 60.5°$$

d) Normalvektor $\vec{BC}\times\vec{BS} = \begin{pmatrix}-8\\4\\-8\end{pmatrix}\times\begin{pmatrix}5\\-4\\-16\end{pmatrix} = \begin{pmatrix}-96\\-168\\12\end{pmatrix} = 12\cdot\begin{pmatrix}-8\\-14\\1\end{pmatrix}$
Lot von L auf BCS: $\vec{r} = \begin{pmatrix}12\\-20\\27\end{pmatrix} + t\begin{pmatrix}-8\\-14\\1\end{pmatrix}$
Durchstosspkt: $\begin{pmatrix}12-8t-5\\-20-14t-1\\27+t-4\end{pmatrix}\cdot\begin{pmatrix}-8\\-14\\1\end{pmatrix} = \begin{pmatrix}7-8t\\-21-14t\\23+t\end{pmatrix}\cdot\begin{pmatrix}-8\\-14\\1\end{pmatrix} = 261 + 261t \overset{!}{=} 0$
$t = -1$
Spiegelbild L': $\vec{r}_{L'} = \vec{r}_L - 2\cdot\begin{pmatrix}-8\\-14\\1\end{pmatrix} = \begin{pmatrix}28\\8\\25\end{pmatrix}$
reflektierter Strahl: $\vec{r} = \vec{r}_R + t\cdot\vec{L'R} = \begin{pmatrix}4\\1\\-4\end{pmatrix} + t\cdot\begin{pmatrix}-24\\-7\\-29\end{pmatrix} \overset{!}{=} \begin{pmatrix}0\\y\\z\end{pmatrix}$
$t = \frac{1}{6}$
$T(0/-\frac{1}{6}/-\frac{53}{6})$

4. Extremalaufgabe

a)



ges: Minimum von $f(x) = \frac{x}{8} + \frac{l}{1}$

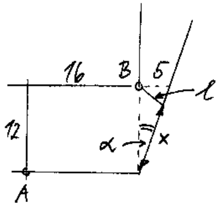
$$= \frac{x}{8} + \sqrt{(16-x)^2+12^2}$$

mit TI-89:

solve (d(...,x) = 0, x) : $x \approx 14.48$

$\approx \underline{\underline{14.5\ m}}$

b)



$\alpha = \arctan \frac{5}{12}$ ($\approx 22.62°$)

$l = \sqrt{12^2 + x^2 - 2 \cdot 12 \cdot x \cdot \cos\alpha}$

ges: Minimum von $f(x) = \frac{16+x}{8} + \frac{l}{1}$

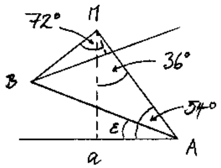
mit TI-89:

solve (d(...,x) = 0, x) : $x \approx 10.49$

$\approx \underline{\underline{10.5\ m}}$

c) Totalreflektion

5. Trigonometrie



oBdA: Seitenlänge $a = 1$

$\sin 36° = \frac{1}{2 \cdot \overline{MA}}$, $\overline{MA} = \frac{1}{2 \cdot \sin 36°}$

$\sin 72° = \frac{1}{2 \cdot \overline{MB}}$, $\overline{MB} = \frac{1}{2 \cdot \sin 72°}$

$$\overline{MB}^2 = \overline{AB}^2 + \overline{AM}^2 - 2 \cdot \overline{AB} \cdot \overline{AM} \cdot \cos \measuredangle BAM$$

$$\frac{1}{4\sin^2 72°} = 1 + \frac{1}{4\sin^2 36°} - 2 \cdot \frac{1}{2 \cdot \sin 36°} \cdot \cos \measuredangle BAM$$

mit TI-89: $\measuredangle BAM \approx 31.72°$

$\varepsilon = 54° - \measuredangle BAM \approx 22.28° \approx \underline{\underline{22.3°}}$

6. Eingespannte kubische Splines

$$\begin{aligned}
f_1 &: y = a_1x^3 + b_1x^2 + c_1x + d_1 \\
f_2 &: y = a_2x^3 + b_2x^2 + c_2x + d_2 \\
f_3 &: y = a_3x^3 + b_3x^2 + c_3x + d_3 \\
f_4 &: y = a_4x^3 + b_4x^2 + c_4x + d_4
\end{aligned}$$

Gleichungen an f:

$$\begin{array}{lll}
A & -125a_1 + 25b_1 - 5c_1 + d_1 & = 0 \\
B & -27a_1 + 9b_1 - 3c_1 + d_1 & = 3.5 \\
B & -27a_2 + 9b_2 - 3c_2 + d_2 & = 3.5 \\
C & d_2 & = 3.5 \\
C & d_3 & = 3.5 \\
D & 8a_3 + 4b_3 + 2c_3 + d_3 & = 3 \\
D & 8a_4 + 4b_4 + 2c_4 + d_4 & = 3 \\
E & 64a_4 + 16b_4 + 4c_4 + d_4 & = 0
\end{array}$$

Gleichungen an f':

$$\begin{array}{lll}
A & 75a_1 - 10b_1 + c_1 & = 0 \\
B & 27a_1 - 6b_1 + c_1 & = 27a_2 - 6b_2 + c_2 \\
C & c_2 & = c_3 \\
D & 12a_3 + 4b_3 + c_3 & = 12a_4 + 4b_4 + c_4 \\
E & 48a_4 + 8b_4 + c_4 & = 0
\end{array}$$

Gleichungen an f'':

$$\begin{array}{lll}
B & -18a_1 + 2b_1 & = -18a_2 + 2b_2 \\
C & 2b_2 & = 2b_3 \\
D & 12a_3 + 2b_3 & = 12a_4 + 2b_4
\end{array}$$

mit TI-89:

$\underline{\underline{f_1(x) = 0.473x^3 - 5.27x^2 - 17.3x - 13.6}}$

$\underline{\underline{f_2(x) = 0.160x^3 + 0.426x^2 - 0.165x + 3.5}}$

$\underline{\underline{f_3(x) = -0.234x^3 + 0.426x^2 - 0.165x + 3.5}}$

$\underline{\underline{f_4(x) = 0.432x^3 - 3.57x^2 + 7.83x - 1.83}}$