Lösungen zur Mathematik-Berufsmatura 3C 2005

1. a) $\begin{array}{ll} \text{I} & 2x+3y-3z=2 \\ \text{II} & x+3y-2z=4 \\ \text{III} & -3x+8y-5z=-42 \end{array}$ $\Rightarrow$ $\begin{array}{ll} 2\text{II}-\text{I} & 3y-z=6 \\ 3\text{II}+\text{III} & 17y-11z=-30 \end{array}$

$11\cdot(2\text{II}-\text{I})-(3\text{II}+\text{III}):\ 16y=96$

$y=6,\ z=12,\ x=10:\quad \mathbb{L}=\underline{\{(10/6/12)\}}$

b) $16x^2+8x-47=0$

$x_{1,2}=\dfrac{-8\pm\sqrt{64+4\cdot16\cdot47}}{32}=\dfrac{-8\pm\sqrt{3072}}{32}$

$=\dfrac{-8\pm32\sqrt{3}}{32}=-\dfrac{1}{4}\pm\sqrt{3}$

$\mathbb{L}=\underline{\{-1.98,\ 1.48\}}$

c) $\ln 2+x\cdot\ln 5.2=\ln 3+(x-1)\ln 0.75$

$x(\ln 5.2-\ln 0.75)=\ln 3-\ln 0.75-\ln 2$

$=\ln 3-(\ln 3-\ln 4)-\ln 2=\ln 4-\ln 2$

$=\ln 2^2-\ln 2=2\cdot\ln 2-\ln 2=\ln 2$

$x=\dfrac{\ln 2}{\ln 5.2-\ln 0.75}$

$\mathbb{L}=\underline{\{0.358\}}$

2. a) $p\cap g\rightarrow T$:

$x^2+1=3x+q$

$x^2-3x+(1-q)=0$

$D=9-4\cdot1\cdot(1-q)$

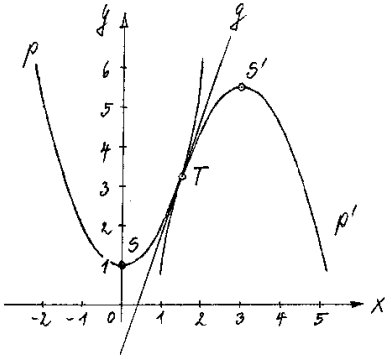
$0\overset{!}{=}5+4q$

$q=\underline{-\dfrac{5}{4}}$

b) Scheitelpunkt $S(0/1)$

Berührungspunkt $T(x/y)$: $x=-\dfrac{-3\pm0}{2\cdot1}=\dfrac{3}{2}$

$y=\left(\frac{3}{2}\right)^2+1=\dfrac{13}{4}$ $\Rightarrow$ $T\left(\frac{3}{2}/\frac{13}{4}\right)$



c) $S'(3/\frac{11}{2})$

$y=-(x-3)^2+\frac{11}{2}$

p': $\underline{y=-x^2+6x-\frac{7}{2}}$

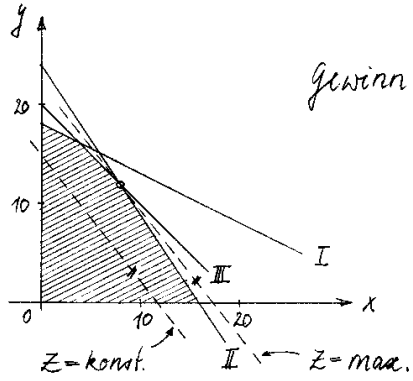
3. Anzahl Rosen pro Woche: x

Anzahl Lilien " " : y

I $\quad \frac{1}{6}x+\frac{1}{3}y\le 6$, also $y\le-\frac{1}{2}x+18$

II $\quad \frac{3}{4}x+\frac{1}{2}y\le 12$, $\quad y\le-\frac{3}{2}x+24$

III $\quad x+y\le 20$, $\quad y\le-x+20$



Gewinn $Z=(3-0.75)x+(2.5-0.5)y$

$=\frac{9}{4}x+2y$

$y=-\frac{9}{8}x+\frac{1}{2}\cdot Z$

Optimum bei $\text{II}\cap\text{III}$: $-\frac{3}{2}x+24=-x+20$

$x=8,\ y=-8+20=12$

Anzahl Rosen: $\underline{8}$

Anzahl Lilien: $\underline{12}$

Gewinn: $Z=\frac{9}{4}\cdot8+2\cdot12=\underline{42\text{ Fr.}}$

4. $f(t) = 2'056'873 \cdot a^t$

a) $f(6) = 2'056'873 \cdot a^6 = 500'000$

$$a = \sqrt[6]{\frac{500'000}{2'056'873}} \approx 0.790$$

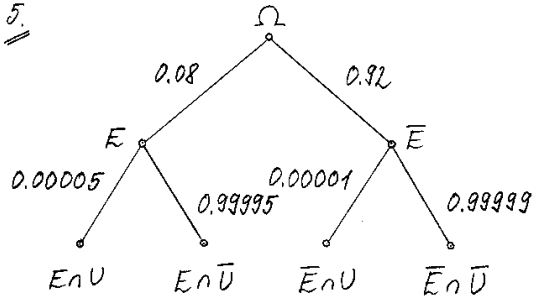
Die jährliche Abschreibung beträgt 21%.

b) $f(10) \approx 2'056'873 \cdot 0.790^{10} \approx 194'750$ Fr.

c) $a^t = \frac{1}{2}$

$$t \approx \frac{-\ln 2}{\ln 0.790} \approx 2.94 \text{ Jahre} \approx 2 \text{ Jahre } 11 \text{ Monate}$$

5.



E = „Es kommt zur Bildung von Eisglätte"

U = „Ein Unfall ereignet sich"

$$P[E|U] = \frac{P[E \cap U]}{P[U]} = \frac{P[U|E] \cdot P[E]}{P[U|E] \cdot P[E] + P[U|\bar{E}] \cdot P[\bar{E}]}$$

$$= \frac{0.000'004}{0.000'004 + 0.000'0092} \approx 30.3\%$$

6. a) i) $\frac{\binom{5}{5}\binom{45}{0}}{\binom{50}{5}} \approx 0.000'047'2\%$

ii) $\frac{\binom{5}{4}\binom{45}{1}}{\binom{50}{5}} \approx 0.0106\%$

iii) $\frac{\binom{5}{3}\binom{45}{2}}{\binom{50}{5}} \approx 0.467\%$

iv) 0.478%

v) $\frac{\binom{5}{2}\binom{45}{3}}{\binom{50}{5}} \cdot \frac{\binom{2}{2}\binom{7}{0} + \binom{2}{1}\binom{7}{1}}{\binom{9}{2}} \approx 2.79\%$

vi) $\frac{\binom{5}{1}\binom{45}{4}}{\binom{50}{5}} \cdot \frac{\binom{2}{2}\binom{7}{0}}{\binom{9}{2}} \approx 0.977\%$

b) 4.25%

c) X = Anzahl Gewinne $\sim Bin(10, 4.25\%)$

i) $P[X=0] \approx (1-0.0425)^{10} \approx 64.8\%$

ii) $P[X=1] \approx \binom{10}{1} \cdot 0.0425^1 \cdot (1-0.0425)^9 \approx 28.7\%$

iii) $P[X \geq 1] = 1 - P[X=0] \approx 35.2\%$