

Goniometrie

Lösungen der Übungsaufgaben

$$1. \quad (a) \quad \sin 15^\circ = \frac{\sqrt{6} - \sqrt{2}}{4} = \frac{\sqrt{2}(\sqrt{3} - 1)}{4} \quad \tan 15^\circ = \frac{1 - \frac{\sqrt{3}}{3}}{1 + \frac{\sqrt{3}}{3}} = 2 - \sqrt{3}$$

$$\cos 15^\circ = \frac{\sqrt{6} + \sqrt{2}}{4} = \frac{\sqrt{2}(\sqrt{3} + 1)}{4} \quad \cot 15^\circ = \frac{\sqrt{3} + 1}{\sqrt{3} - 1} = 2 + \sqrt{3}$$

$$(b) \quad \sin 75^\circ = \frac{\sqrt{2}(\sqrt{3} + 1)}{4} \quad \tan 75^\circ = 2 + \sqrt{3}$$

$$\cos 75^\circ = \frac{\sqrt{2}(\sqrt{3} - 1)}{4} \quad \cot 75^\circ = 2 - \sqrt{3}$$

$$(c) \quad \sin 105^\circ = \frac{\sqrt{2}(\sqrt{3} + 1)}{4} \quad \tan 105^\circ = -2 - \sqrt{3}$$

$$\cos 105^\circ = -\frac{\sqrt{2}(\sqrt{3} - 1)}{4} \quad \cot 105^\circ = -2 + \sqrt{3}$$

$$2. \quad (a) \cos \alpha \quad (b) -\sin \alpha \quad (c) \sqrt{2} \sin \alpha \quad (d) \sqrt{2} \cos \alpha$$

$$3. \quad (a) \tan \alpha \quad (b) -\tan \alpha \quad (c) \frac{\tan \beta}{\tan \alpha} = \cot \alpha \tan \beta \quad (d) -\tan \alpha \tan \beta$$

4.

$$\begin{aligned} \tan \alpha + \cot \alpha &= \frac{\sin \alpha}{\cos \alpha} + \frac{\cos \alpha}{\sin \alpha} = \frac{\sin^2 \alpha + \cos^2 \alpha}{\sin \alpha \cos \alpha} = \frac{1}{\sin \alpha \cos \alpha} \\ &= \frac{2}{2 \sin \alpha \cos \alpha} = \frac{2}{\sin(2\alpha)} \end{aligned}$$

5.

$$\begin{aligned} \tan(45^\circ + \alpha) - \tan(45^\circ - \alpha) &= \frac{\tan 45^\circ + \tan \alpha}{1 - \tan 45^\circ \tan \alpha} - \frac{\tan 45^\circ - \tan \alpha}{1 + \tan 45^\circ \tan \alpha} \\ &= \frac{1 + \tan \alpha}{1 - \tan \alpha} - \frac{1 - \tan \alpha}{1 + \tan \alpha} \\ &= \frac{(1 + \tan \alpha)^2 - (1 - \tan \alpha)^2}{1 - \tan^2 \alpha} \\ &= \frac{1 + 2 \tan \alpha + \tan^2 \alpha - 1 + 2 \tan \alpha - \tan^2 \alpha}{1 - \tan^2 \alpha} \\ &= 2 \frac{2 \tan \alpha}{1 - \tan^2 \alpha} = 2 \tan(2\alpha) \end{aligned}$$

6.

$$\begin{aligned} (a) \quad \sin 15^\circ &= \sqrt{\frac{1 - \cos 30^\circ}{2}} = \sqrt{\frac{1 - \frac{\sqrt{3}}{2}}{2}} = \sqrt{\frac{2 - \sqrt{3}}{4}} = \sqrt{\frac{4 - 2\sqrt{3}}{8}} \\ &= \sqrt{\frac{3 - 2\sqrt{3} + 1}{8}} = \sqrt{\frac{(\sqrt{3} - 1)^2}{8}} = \frac{\sqrt{3} - 1}{2\sqrt{2}} = \frac{\sqrt{2}(\sqrt{3} - 1)}{4} \end{aligned}$$

$$(b) \cos 15^\circ = \sqrt{\frac{2 + \sqrt{3}}{4}} = \frac{\sqrt{2}(\sqrt{3} + 1)}{4}$$

$$(c) \tan 15^\circ = \frac{1 - \cos 30^\circ}{\sin 30^\circ} = \frac{1 - \frac{\sqrt{3}}{2}}{\frac{1}{2}} = 2 - \sqrt{3}$$

$$(d) \cot 15^\circ = \frac{1}{\tan 15^\circ} = \frac{1}{2 - \sqrt{3}} = 2 + \sqrt{3}$$

$$7. \quad (a) \ s_{12} = r\sqrt{2 - \sqrt{3}} \quad (b) \ h_{12} = \frac{r}{2}\sqrt{2 + \sqrt{3}} \quad (c) \ A_{12} = \frac{1}{4}r^2$$

$$8. \tan 7.5^\circ = \sqrt{6} - \sqrt{3} + \sqrt{2} - 2 = 2\sqrt{2 + \sqrt{3}} - 2 - \sqrt{3}$$

$$9. \quad (a) \tan \frac{\alpha + \beta}{2} \quad (b) -\cot \frac{\alpha + \beta}{2}$$

$$10. \quad (a) \frac{1}{2}\sqrt{6} \quad (b) -\frac{1}{2}\sqrt{2}$$

11. siehe Aufgabe 1.